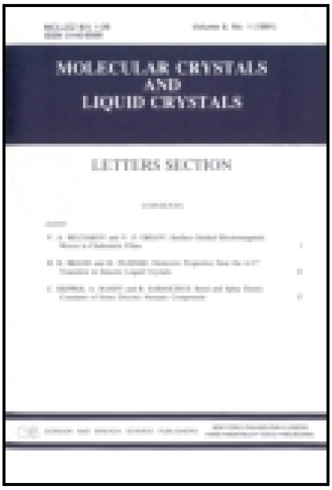

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Phase Retardation Difference of Liquid Crystal Cells with Symmetric and Asymmetric Boundary Conditions

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Phase Retardation Difference of Liquid Crystal Cells with Symmetric and Asymmetric Boundary Conditions

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Phase retardation $\Delta\Phi$ values for the LC cells with homogeneous and inhomogeneous LC director distribution vs. *the LC pretilt angle θ_0 on the cell's substrates has been simulated. Cases of symmetric and hybrid cells with both positive and negative optical anisotropy LC with θ_0 values in the range from 0 to 90° and $\theta_0 > 90°$ are reviewed.*

Keywords Liquid crystals; homogeneous and inhomogeneous orientation; optical anisotropy; pretilt angle

1. Introduction

In many types of to-date liquid crystal displays (LCD) tilted alignment and/or sophisticated configuration of the LC director is used (MVA, OCB et al. [1–3]). It can improve the LCD speed of response or viewing angle range [4]. Therefore investigation of optical properties of such cells, in particular, determination of an influence of the director distribution on the polarized ray propagation through a birefringent material like the LC is an actual task. This effect is described by a value of the phase retardation $\Delta\Phi$ between the polarized extraordinary and ordinary waves passing through the LC cell. It is well investigated for the cases of homogeneous planar or vertical alignment or TN-mode. However there is no satisfactory data for the LC cells with inhomogeneous LC director distribution. The goal is to calculate the $\Delta\Phi$ values for the LC cells with homogeneous and inhomogeneous LC director distribution *vs* the LC pretilt angle θ_0 on the cell's substrates.

In our previous publications [5, 6] we have received dependences of the phase retardation difference $\Delta\Phi$ on the pretilt angle θ_0 that have been calculated for the LC cells with homogeneous and inhomogeneous LC director configuration with symmetric and asymmetric boundary conditions. The single constant approximation (the Frank elastic constants for a nematic LC $K_{11} = K_{33}$) was used to simplify the calculations. In this case, the LC elastic energy is independent on the local tilt angle and the tilt angle variation inside the cell is described by a linear function for every LC director configuration [5]. We have changed the pretilt angle θ_0 in the range from 0 to 90° that corresponds to the case of the LCD practical applications.

Besides of the LC director configurations typical for usual nematic LC cells (e.g., splay for $\theta_0 < 45°$ or bend for $\theta_0 > 45°$) we have considered many configurations that are nor realized in the cells owing to Franck elastic forces (e.g., splay for $\theta_0 > 45°$ or bend

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for $\theta_0 < 45°$). However they can appear during dynamic switching (e.g., OCB- or pi-cells [2, 3]). Such configurations can be also created by using layered media that consists of alternating layers of birefringent material, e.g., birefringent polymeric LC material [7, 8].

In this paper we present simulation results of $\Phi(\theta_0)$ dependences for different types of birefringent films with inhomogeneous spatial distribution of the material birefringence. Following cases are considered:

- Nematic LC cells with symmetric pretilt angles on opposite substrates, positive and negative birefringence, the pretilt angle $0 < \theta_0 < 90°$
- Nematic LC cells with asymmetric pretilt angles on opposite substrates (hybrid cells), the pretilt angle $0 < \theta_0 < 90°$.
- Nematic birefringent cells with the pretilt angle $\theta_0 > 90°$.

2. Results

For all the tasks presented in the paper the phase retardation $\Delta\Phi$ for the cells with arbitrary LC director distribution is described with the expression as follows:

$$\Delta\Phi = \frac{2\pi}{\lambda}\left[\int_0^L \frac{n_o n_e dz}{(n_o \cos^2\theta(z) + n_e \sin^2\theta(z))^{1/2}} - n_o L\right] \quad (1)$$

where L is the cell thickness, λ is the wavelength, n_o and n_e are the refractive indices for the ordinary and extraordinary rays, accordingly, $\theta(z)$ is the LC director distribution within the cell.

The case of the light normal incidence and the single constant approximation (nematic LC Franck elasticity coefficients $K_{11} = K_{33}$) is considered.

A phase difference parameter $\Phi = \Delta\Phi/\Delta\Phi_{max}$ was introduced in [5]. The phase retardation difference $\Delta\Phi$ is reduced to its maximum value $\Delta\Phi_{max} = 2\pi\Delta nL/\lambda$, $\Delta n = n_e - n_o$ is the LC birefringence. The case of $\Delta\Phi_{max}$ is realized for the cells with the LC planar alignment ($\theta_0^{(1)} = \theta_0^{(2)} = 0$).

Dependences of the phase retardation difference $\Delta\Phi$ on the pretilt angle θ_0 have been numerically calculated for the LC cells with homogeneous, splay or bend director configuration (Fig. 1) [5]. In the splay and bend cells a case of symmetric boundary conditions (the pretilt angle had the same values with opposite sign at opposite substrates) was realized. Typical Φ dependences on the pretilt angle θ_0 in the cells with splay (S), homogeneous (H) and bend (B) LC orientation for different n_e and n_o values are presented in Fig. 2. In [5] analytical approximations have been received for the cases of $\theta_0 << 1$, $\theta_0 \to \pi/2$, $\theta_0 \sim \pi/4$.

In the paper presented dependences of Φ *vs.* θ_0 ($0 < \theta_0 < 90°$) for $n_o = 1.5$ and $n_e = 1.3; 1.35; 1.4; 1.45$ ($\Delta n < 0$) have been calculated (Figs. 3 and 4). The $\Phi(\theta_0)$ dependences for the LC with negative optical anisotropy have approximately the same view like the same curves for the LC with $\Delta n > 0$ [5] but they are characterized with larger Φ value that increases when n_e reduces. It is very good to see in the quasilinear part of the $\Phi(\theta_0)$ dependence in the vicinity of $\theta_0 \approx \pi/4$ (Fig. 4).

The linear part of the curves shown in Fig. 4 is described by an equation as follows:

$$\Phi(\theta_0) = \frac{1}{4}\left[(1 + \pi/2 - 2\theta_0)\left(3 - \frac{n_e}{n_o}\right)\right] = \frac{1}{2}\left[(1 + \pi/2 - 2\theta_0)\left(1 - \frac{\Delta n}{2n_o}\right)\right] \quad (2)$$

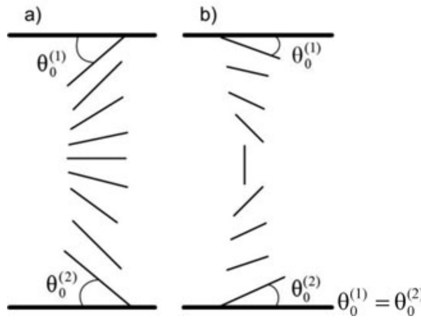

Figure 1. LC cells with symmetric pretilt angles ($\theta_0^{(1)} = \theta_0^{(2)}$) on opposite substrates. S left and B right [5].

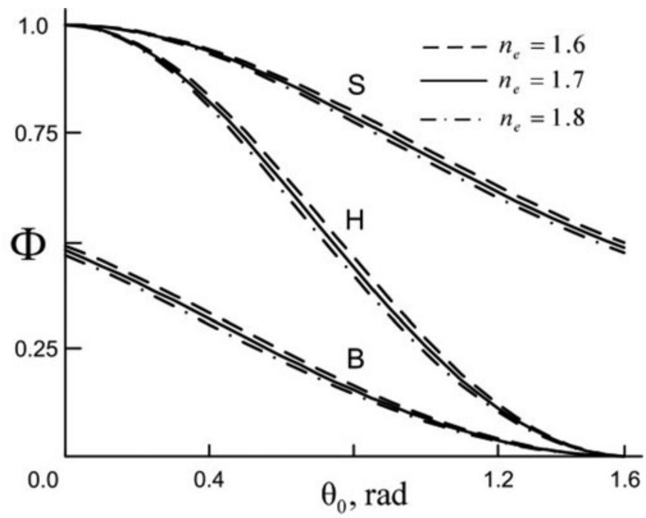

Figure 2. Dependence of the reduced phase difference Φ on the pretilt angle θ_0 in the cells with splay (upper curves), homogeneous (middle curves) and bend (lower curves) LC orientation for different n_e values. $n_o = 1.5$.

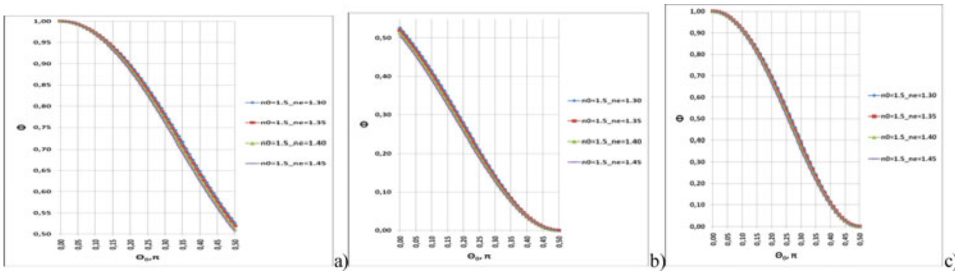

Figure 3. $\Phi(\theta_0)$ dependences for the S (a), B (b) and H (c) cells with negative optical anisotropy.

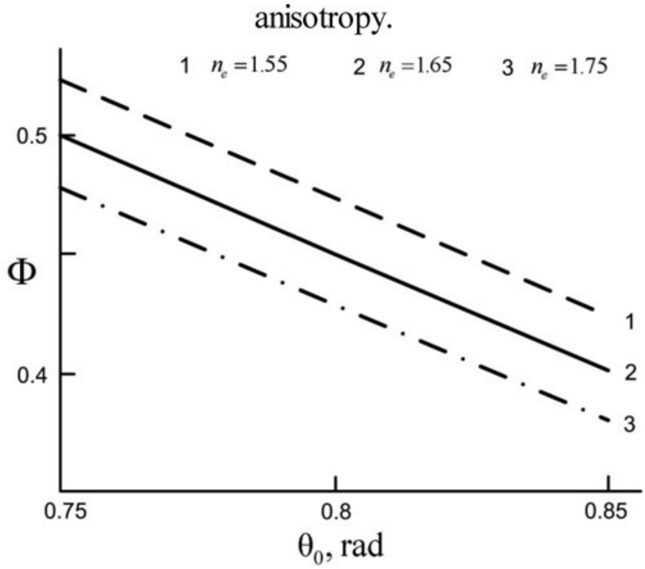

Figure 4. $\Phi(\theta_0)$ dependences for the H cells in the vicinity of $\theta_0 \approx \pi/4$.

The $\Phi(\theta_0, x)$ dependence has no discrepancy at $\Delta n = 0$ (x = 1), it is a continuous function if birefringence changes its sign. It is valid for all three geometries of the LC cells investigated in this paper.

Three types of hybrid LC cells with asymmetric pretilt angles on opposite substrates have been considered (Fig. 5). Further in the text such abbreviations will be used: Hyb90°, HybS, and HybB. In every LC director configuration the boundary angles $\theta_0^{(1)}$, $\theta_0^{(2)}$ vary within the range from 0 to 90°. For the first geometry (Hyb90°) the total tilt angle θ(z) changes in the same range. However in the geometries HybS and HybB the total change of the tilt angle θ(z) can achieve a value of 180°. Such case is hypothetical for usual nematic cells, but it can be realized by using polymeric birefringent media. In the HybS configuration the θ(z) goes through the zero value, and the director distribution looks like splay. In the HybB configuration the θ(z) goes through the value of 90°, and the director distribution looks like bend. The tilt angle variation inside the cell is described by a linear function for every LC director configuration.

In the case of the Hyb90° cell (Fig. 6) the total change is of order of 0.5. A slight difference from 0.5 is determined by the n_e value. The upper and lower curves in right part of Fig. 2 are asymmetric in relation to the $\Phi(\theta_0^{(1)} = \theta_0^{(2)})$ $\theta_0^{(2)} = 0.25\pi$ curve that corresponds to parameter $\theta_0^{(2)} = 0.25\pi$. These curves correspond to the cases of "classical" hybrid LC alignment on opposite substrates (zero tilt on one substrate and vertical alignment on another one). If a line for the case of the dependence $\Phi(\theta_0^{(1)} = \theta_0^{(2)})$ will be drawn it will coincide with an appropriate line for the $\Phi(\theta_0)$ dependence calculated in [5] for the same refractive indices. The dependence will change from 1 to 0 at increasing θ_0 value. The slope of $\Phi(\theta_0^{(1)}, \theta_0^{(2)})$ curves at $\theta_0 = 0.25\pi$ is around one half of the $\Phi(\theta_0)$ dependence slope for the case of homogeneous director distribution in [5].

Main difference of the $\Phi(\theta_0^{(1)})$ dependences behavior for both HybS, HybB cells and Hyb90° cells is observed for $\theta_0^{(2)} \sim 0.5\pi$ (HybS; Fig. 7) and small $\theta_0^{(2)}$ angles (HybB). If $\theta_0^{(2)} = 0.45\pi$ the $\Phi(\theta_0^{(1)})$ dependence for the HybS cell varies from 0.53 to 0.63 and the

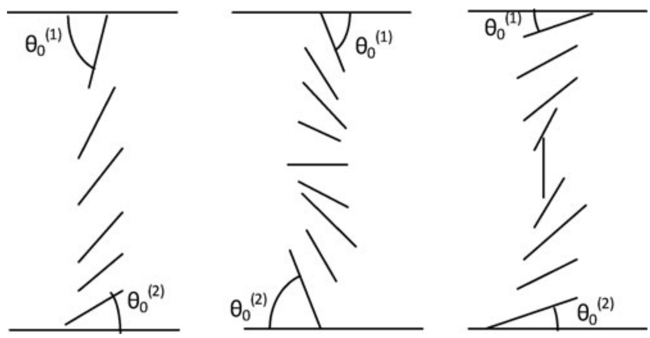

Figure 5. Hybrid LC cells with asymmetric pretilt angles on opposite substrates. From left to right: Hyb90°, HybS, HybB.

curve has its maximum at $\theta_0^{(1)} \to 0.2\pi$. If $\theta_0^{(2)} = 0.45\pi$ the $\Phi(\theta_0^{(1)})$ dependence for the HybB cell varies from 0.37 to 0.48 and the curve has its minimum at $\theta_0^{(1)} \to 0.3\pi$. For both types of the cells $\Phi(\theta_0^{(1)} = 0) \approx \Phi(\theta_0^{(1)} = 0.5\pi)$ because the z-coordinate corresponding to $\theta(z) = 0$ (HybS) or $\theta(z) = 0.5\pi$ (HybB) shifts from the center.

We mentioned above that in typical LC cells with static driving $0 \le \theta_0^{(2)} \le \pi/2$. That means $\theta(z)$ variation in the cell without intermediate vertical LC orientation in the cells with splay director distribution and intermediate planar orientation in the cells with bend distribution. However configurations with $\theta_0 > \pi/2$ may arise in dynamically driven LC cells. If in twist or initially planar aligned cells high voltage ($U >> U_{th}$) is applied the LC alignment in the middle part of the LC cell is almost vertical. When the voltage is rapidly switched off the LC director in the cell's middle part reorients in the reverse direction in relation to the orientation direction during switching on cycle. The back flow results in appearance of the LC tilt angles exceeding $\pi/2$ [9, 10]. Besides the LC director distribution with $\theta > \pi/2$ can take place in LC polymers if special actions are made to fix unusual director distribution.

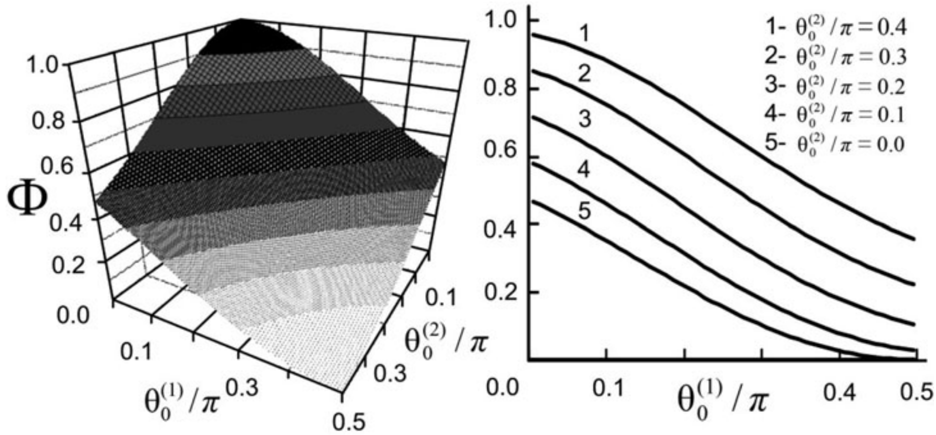

Figure 6. $\Phi(\theta_0^{(1)}, \theta_0^{(2)})$ dependences for the case of Hyb90° cells. Top: Φ behaviour at simultaneous variation of both $\theta_0^{(1)}$ and $\theta_0^{(2)}$. Bottom: $\Phi(\theta_0^{(1)})$ dependences at fixed $\theta_0^{(2)}$ value or *vice versa*.

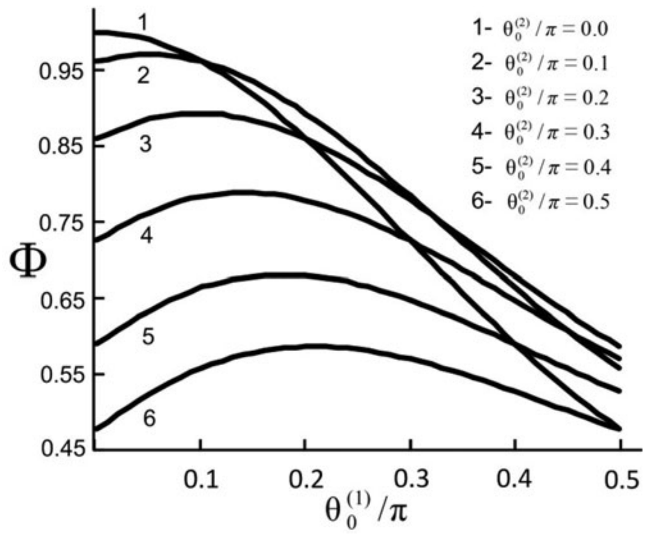

Figure 7. $\Phi(\theta_0^{(1)}, \theta_0^{(2)})$ dependences for the case of HybS cells at fixed $\theta_0^{(2)}$ value.

Let us analyze $\Phi(\theta_0)$ dependences for S- and B-geometries considered for the case $\theta_0 > \pi/2$. An example of the director distribution inside the cell is demonstrated in Fig. 8. Main difference of the director distributions shown in Fig. 1 is appearance of two regions in the cell with intermediate vertical LC orientation in the S-cells and planar orientation in the B-cells. If hypothetical θ_0 value will exceed 180° (π rad) the number of regions with intermediate planar and vertical LC orientation will rise.

In Fig. 9 the $\Phi(\theta_0)$ dependences are shown for H, S- and B-cells' geometries with expanded range of θ_0 values. We see appearance of extrema in the $\Phi(\theta_0)$ dependences. The

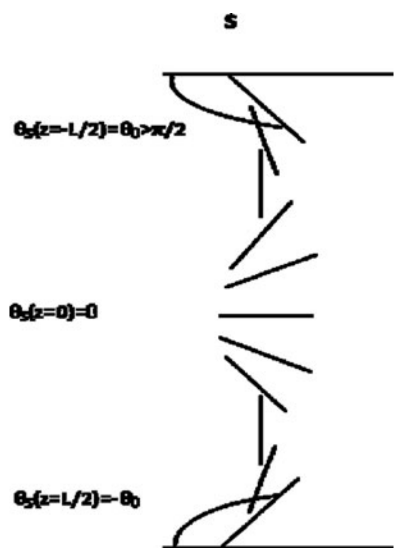

Figure 8. The LC director distribution inside the S cell with $\theta_0 > \pi/2$.

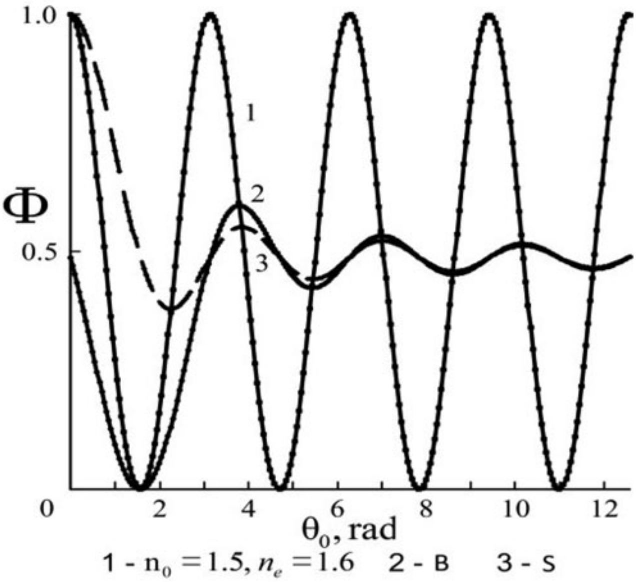

Figure 9. $\Phi(\theta_0)$ dependences for S- (2) and B- (3) geometries in the case of $\theta_0 > \pi/2$. 1 – homogeneous orientation.

curve for S-configuration has its first minimum $\Phi(\theta_0) = 0.35 - 0.45$ at $\theta_0 = 120^\circ - 125^\circ$. The minimum value and its position depend on refractive indices. For B-configuration the $\Phi(\theta_0)$ curve has its first maximum $\Phi(\theta_0) \approx 0.65$ at $\theta_0 \approx 120^\circ$. The maximum value and its position depend on refractive indices too. Both types of curves (S- and B-) trend to $\frac{1}{2}$ with infinite growth of θ_0. In this case number and size of the regions with intermediate planar and vertical LC orientation have the same probability.

3. Conclusion

The calculation method developed allows a selection of the LC director configuration to change the LC cell phase retardation that can be used in optical compensator design. The method can be used for different LC cells with a given LC director distribution and symmetric or asymmetric boundary conditions [11–15].

The results can be used also to develop methods of the LC pretilt angle measurement in the cells with sophisticated director configuration. The methods known provide good accuracy for the cells with homogeneous tilt inside the cell. These methods can give wrong pretilt angle value if the director distribution is inhomogeneous because the same $\Phi(\theta_0)$ value can be obtained in an experiment in LC cells with different boundary condition and director distribution [5, 16–18].

4. Acknowledgments

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